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On the Possibility of an Electromagnetic Foundation of Mechanics.

by W. Wien.

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H. A. LORENTZ[1] has recently tried to explain gravitation by electrostatic attractions between the elements of bodies consisting of ions. For that purpose, he makes the assumption that the attractions between positive and negative electricity outweighs the repulsion between electricities of same sign. I was animated by that to publish considerations (made by me already some time ago) on the same subject, and where I even go beyond LORENTZ's standpoint.

It is without doubt one of the most important tasks of theoretical physics, to connect with each other the, at the beginning, completely isolated areas of mechanical and electromagnetic phenomena, and to derive the differential equations applying to any of them from a common foundation. MAXWELL and THOMSON and subsequently BOLTZMANN and HERTZ, have taken the, at the beginning, surely natural way to chose mechanics as the foundation and to derive MAXWELL's equations from it. Numerous analogies existing between electrodynamic and hydrodynamic as well as elastic processes, seemed to repeatedly allude to this way. HERTZ's mechanics appears to be devised in its entire structure, not only to include mechanical, but also electromagnetic phenomena. That a mechanical derivation of MAXWELL's electrodynamics is possible, is known to be shown by MAXWELL himself.

These investigations have without doubt the great merit, to have proven that both areas must be founded by something common, and that the current separation is not founded in the nature of this subject. On the other hand, it seems certain to me from these considerations, that the system of our recent mechanics is inappropriate for the representation of electromagnetic processes.

One will never acknowledge the complicated mechanical models, formed after the machines devised for special technical purposes, as a definitely satisfactory image for the inner composition of the aether.

Whether HERTZ's mechanics, whose structure is indeed appropriate for the inclusion of very general kinematic connections, can render more functional, must remain undecided. For the time being, it was unable to represent even the simplest processes lying outside of kinematics.

The opposite attempt seems to me much more promising as the foundation for further theoretical work, *i.e.*, to consider the electromagnetic fundamental equations as the more general ones, from which the mechanical ones have to be derived.

The actual foundation would be formed by the electric and magnetic polarization in free aether, connected with each other by MAXWELL's differential equations. As to how these equations can be derived best from the facts, is a question with which we don't have to deal with at this place.

If we denote X,Y,Z the components of electric, L,M,N those of the magnetic polarization, A the reciprocal speed of light, x,y,z the rectangular coordinates, we thus have:

$$(1)\qquad \begin{cases} A\frac{\partial X}{\partial t}=\frac{\partial M}{\partial z}-\frac{\partial N}{\partial y}, & A\frac{\partial L}{\partial t}=\frac{\partial Z}{\partial y}-\frac{\partial Y}{\partial z},\\ A\frac{\partial Y}{\partial t}=\frac{\partial N}{\partial x}-\frac{\partial L}{\partial z}, & A\frac{\partial M}{\partial t}=\frac{\partial X}{\partial z}-\frac{\partial Z}{\partial x},\\ A\frac{\partial Z}{\partial t}=\frac{\partial L}{\partial y}-\frac{\partial M}{\partial x}, & A\frac{\partial N}{\partial t}=\frac{\partial Y}{\partial x}-\frac{\partial X}{\partial y}.\end{cases}$$

The electric and magnetic quanta are therefore given as integration constants, when we differentiate equations (1) with respect to x,y,z and then sum up. Then it is namely

$$\frac{\partial}{\partial t}\left(\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}\right)=0,\quad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x}+\frac{\partial M}{\partial y}+\frac{\partial N}{\partial z}\right)=0,$$

thus

$$(2)\qquad \frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}=-4\pi\varsigma,\quad \frac{\partial L}{\partial x}+\frac{\partial M}{\partial y}+\frac{\partial N}{\partial z}=-4\pi m,$$

where ς and m are independent from time, thus being temporally invariable quanta.

If one multiplies the first row of equations (1) with $X / 4\pi, Y / 4\pi, Z / 4\pi$,, the second with $L / 4\pi, M / 4\pi, N / 4\pi$,, and sum up all of them, one thus obtains after partial integration over a closed space, whose surface normal shall be n and surface element be dS, the theorem

$$(3)\qquad \begin{cases} \frac{1}{8\pi}\frac{d}{dt}\int\int\int dx\ dy\ dz\ \left(X^2+Y^2+Z^2+L^2+M^2+N^2\right)\\ =\int dS\left[(YN-ZM)\cos(xn)+(ZL-XN)\cos(ny)+(XM-YL)\cos(nz)\right].\end{cases}$$

If either X,Y,Z or L,M,N vanish at the surface, we thus have

$$(4)\qquad \frac{1}{8\pi}\int\int\int dx\ dy\ dz\ \left(X^2+Y^2+Z^2+L^2+M^2+N^2\right)=const.$$

Those on the left, that are always being constant when summed over a sufficiently great space, we denote as the electromagnetic energy.

Now we make the assumption, that the mechanical processes are of electromagnetic nature as well, *i.e.*, that they can be developed from the foundations considered.

For that purpose, we assume at first, that the substrate denoted as matter is composed of positive and negative electric quanta, namely of such elementary quanta, which we simply have to view as convergence points of electric force lines.

However, we must attribute to such an elementary quantum a certain extension, because otherwise the energy store represented by it, would be infinitely great compared with the quantum itself. Since the entire matter shall be constituted by those quanta, they have to be assumed as so small, so that the atomic weights are whole multiples of them. The positive elementary quantum is furthermore to be viewed as distant from the negative one by a certain small distance.

That matter is composed of such dipoles, is hardly a special assumption, and surely is currently admitted by all physicists. Until now, a ponderable substance was additionally assumed, that we want to identify with these quanta.

The statement, that matter as well as electricity is formed atomistically, is equivalent according to the view held by us.

The aether itself is to be viewed as stationary according to LORENTZ. Spatial changes can only occur with respect to electric quanta, and to speak about a motion of the aether would make no sense by the principles followed here.

All forces are to be reduced to the known electromagnetic ones, *i.e.*, to tensions in the aether in the sense of MAXWELL, although the concept taken from the theory of elasticity is hardly still meaningful here.

At small velocities of the moving quanta, the electrostatic forces are the ones that are effective between the quanta.

Whether a reduction of molecular forces to such forces is possible, must remain undecided yet. It's only clear, that by different arrangements of positive and negative quanta in different distances, one can obtain very complicated effects. By that assumption, one would decrease the difficulty to the theory of stationary aether represented by MICHELSON's interference experiment.

H. A. LORENTZ[2] has alluded to the fact, that the length of a body in the direction of Earth's motion is contracted by the velocity *v* of this motion in the ratio $\sqrt{1-A^2v^2}$, if the molecular forces could be replaced by electrostatic forces.

By that, MICHELSON's result would be explained, when one can neglect molecular motions. As to how far this is true, must be shown by gas-theoretical investigations.

For the explanation of gravitation, following LORENTZ, we must assume two different kinds of electric polarizations. Any of them satisfies for itself MAXWELL's equations. Additionally, with respect to static fields it is given

$$X=-\frac{\partial\phi}{\partial x},\quad Y=-\frac{\partial\phi}{\partial y},\quad Z=-\frac{\partial\phi}{\partial z}$$

and the energy

$$\frac{1}{8\pi}\int\int\int dx\ dy\ dz\ \left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2$$

$$= \frac{1}{8\pi}\int dS \frac{\partial\phi}{\partial n}\phi - \frac{1}{8\pi}\int\int\int dx\ dy\ dz\ \phi\Delta\phi.$$

If φ or $\partial\varphi/\partial$ vanishes at the surface of space, the energy is

$$= -\frac{1}{8\pi}\int\int\int dx\ dy\ dz\ \phi\Delta\phi.$$

Now, according to (2)

$$\Delta\phi = -4\pi\varsigma, \quad \phi = \int\int\int \frac{dx\ dy\ dz\ \varsigma}{r},$$

thus the integral

$$= \frac{1}{2}\int\int\int \frac{dx\ dy\ dz\ \varsigma}{r}\int\int\int dx'\ dy'\ dz'\ \varsigma'$$
$$= \int\int\int\int\int\int \frac{\varsigma\varsigma' dx\ dy\ dz\ dx'\ dy'\ dz'}{r}$$

If two quanta of same sign are located at the distance *r*

$$e = \varsigma\ dx\ dy\ dz,$$
$$e' = \varsigma'\ dx'\ dy'\ dz',$$

then the energy is

(5) $$\frac{ee'}{r} = -\int\limits_{\infty}^{r} \frac{ee'}{r^2}dr;$$

this energy was produced by work against a force acting repulsive between the quanta, of amount

(6) $$-\frac{ee'}{r^2}.$$

By that, the force acting between two quanta is defined.

This law must hold for any of the two polarizations.

If positive and negative quanta start to interact, then LORENTZ's assumption is, that the attracting force occurring, is greater in a certain ratio as the repulsive one between two quanta of equal sign. On greater distances, the dipoles act as if the positive and negative quantum would be located at the same place. Thus, by the total action of negative and positive quanta upon a second dipole, one obtains an excess of attraction.

This explanation of gravitation has the direct consequence, that the disturbance itself is propagating with the speed of light, and it must experience a modification by the motion of the mutually attracting bodies. LORENTZ has investigated, whether this modification of gravitation can explain the anomalies of the motion of mercury, yet he found a negative result. Some astronomers believed that it is necessary to assume a greater speed than that of light for the propagation of gravitation. However, one cannot speak about a propagation speed of gravitation (as a static force) itself.

This would only make sense, when one was able to increase or decrease gravitation, and to observe the disturbances caused by that.

However, since gravitation always acts invariably, only the extraordinary small changes caused by motion can come into question, which are of second order as shown by LORENTZ.

The inertia of matter, which represents the second independent definition of matter besides gravitation, can be derived without further hypotheses from the concept of electromagnetic inertia, which is already employed many times.

We think of the electric elementary quantum as an electrified point. The forces and polarizations emanating from such a moving point, were derived by HEAVISIDE.[3]

Since equal amounts of positive and negative quanta are always moving together, thus, in a distance great against the their mutual distance, the forces emanating from them and the polarizations are neutralizing each other, except gravitation discussed before. Though we will assume in the following, that the extension of the quanta is so small compared to their distance, that the energy of any of them is so great, as if the second one would not be present.

According to a calculation by SEARLE,[4] those polarizations emanate from an ellipsoid moving in the direction of its axis *a* with velocity *v*, whose other two axes are $a/\sqrt{1-A^2v^2}$, and which carries the same charge upon its surface. The ratio of the axes therefore depends on the velocity.

The energy of such an ellipsoid is according to SEARLE

$$E=\frac{e^2}{2a}\left(1+\frac{1}{3}A^2v^2\right).$$

The ellipsoid with the same axes, has the energy in a state of rest

$$\mathfrak{E}=\frac{e^2\sqrt{1-A^2v^2}}{2a\quad Av}\arcsin Av.$$

Naturally, the energy $\mathfrak{E}$ of the resting ellipsoid, may not contain the velocity *v*.

Thus, since *e* is invariable, *a* is variable:

$$2a=\frac{e^2\arcsin Av\sqrt{1-A^2v^2}}{Av\mathfrak{E}},$$

$$E=\mathfrak{E}\frac{Av\left(1+\frac{1}{2}A^2v^2\right)}{\sqrt{1-A^2v^2}\arcsin Av}$$

or by series expansion

(7) $$E = \mathfrak{E}\left(1 + \frac{2}{3}A^2v^2 + \frac{16}{45}A^4v^4 \dots\right).$$

The energy increase caused by motion, is thus in first approximation

$$\frac{2}{3}\mathfrak{E}A^2v^2 = \frac{m}{2}v^2,$$

thus the inertial mass $m = \frac{4}{3}\mathfrak{E}A^2$.

By that, the mass defined by inertia would only be constant at small velocities, and would increase with increasing velocity. Since inertia as well as gravitation emerging from the body, is proportional to the number of quanta composing the body, it follows, that the mass defined by inertia must be proportional to the one specified by gravitation. If a body with mass $m = \frac{4}{3}\mathfrak{E}A^2$, is attracted by a body of mass *M* up to a distance of *r*, then the electromagnetic energy store of gravitation is diminished by the amount $\epsilon\frac{4}{3}\mathfrak{E}A^2M/r$, where ε denotes the gravitational constant.

This energy is transformed into kinetic energy to produce the velocity *v*. Thus we have

$$\frac{2}{3}\mathfrak{E}A^2v^2\left(1 + \frac{8}{15}A^2v^2 \dots\right) = \epsilon\frac{\frac{4}{3}\mathfrak{E}A^2M}{r},$$

or, since *v* = *dr* / *dt*

(8) $$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{\epsilon M}{r}\left(1 - \frac{8}{15}A^2\left(\frac{dr}{dt}\right)^2\right).$$

Instead of it, we can write

(9) $$\frac{1}{2}\left(\frac{dr}{dt}\right)^2\left(1 + \frac{16}{15}A^2\frac{\epsilon M}{r}\right) = \frac{\epsilon M}{r}.$$

If the masses *M* and *m* would attract each other according to *Weber*'s law, one would have

$$m\frac{d^2r}{dt^2} = -\frac{\epsilon mM}{r}\left\{1 - \frac{A^2}{2}\left(\frac{dr}{dt}\right)^2 + rA^2\frac{d^2r}{dt^2}\right\}.$$

If we multiply with *dr* / *dt* and then integrate, we have:

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{\epsilon M}{r}\left[1 - \frac{A^2}{2}\left(\frac{dr}{dt}\right)^2\right],$$

where the integration constant is so specified, that the body is at rest in infinite distance.

If we write this equation

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2\left(1+A^2\frac{\epsilon M}{r}\right)=\frac{\epsilon M}{r}, \tag{10}$$

then it agrees with equation (9) up to a factor $\frac{16}{15}$ instead of 1. By consideration of the second approximation for inertia, we thus approximately obtain the same action between the two masses, as if the masses themselves would be invariable, but WEBER's law would hold instead of NEWTON's law.

It's known, that WEBER's law was applied with some success to the theory of molecular motion.

A precise test of these investigations, and an extension by applying it to other, fast moving celestial bodies with strong eccentric path, would lead us to a comparison of our results with experience. Though it is to be considered here, that new terms of same order are added by the motion in a curved path. For a body in elliptical path, the calculation would thus to be supplemented.

Only have with respect to cathode rays we have such great velocities required, so that the square of velocity multiplied with the reciprocal speed of light, is not getting too small.

The fastest, recently produced rays have $\frac{1}{3}$ of the speed of light. Here, the apparent increase of mass would be ca. 7 perc.; the slightest velocity is $\frac{1}{30}$ of the speed of light,[5] the corresponding increase of mass would amount here only 0,07 perc. An increase of mass compared to the electric charge at cathode rays of great velocity, is indeed contained in LENARD's observations.[6] Though the differences found by LENARD are much too great, to find their explanation in the electromagnetic inertia.

However, these quantitative measurements are still not to be seen as decisive.

If we confine ourselves to small velocities, then we have the same expression for the kinetic energy, that is stated by mechanics for the living force. The magnitude of acceleration, however, cannot be derived from that without further ado.

Acceleration presupposes a variability of velocity. The expressions for electromagnetic energy, however, are only derived under the assumption of a time-independent value of velocity.

For variable velocities, the problem of a moving electric quantum is not rigorously solved thus far.

Though we obtain from MAXWELL's equations a criterion concerning the magnitude of the error made by us when we apply the expressions for energy to variable velocities as well.

The electric and magnetic polarizations are in our case, when the motion occurs in the direction *x*,

$$X=\frac{\partial U}{\partial x}\left(1-A^2v^2\right),\quad Y=\frac{\partial U}{\partial y},\quad Z=\frac{\partial U}{\partial z},$$

$$M=-Av\frac{\partial U}{\partial z},\quad N=Av\frac{\partial U}{\partial y},\quad L=0,$$

$$U=\frac{e}{\sqrt{r^2-A^2v^2\varsigma^2}},\quad \varrho^2=x^2+y^2.$$

Here, the coordinate system is rigidly connected with the moving point.

These expressions satisfy MAXWELL's equations, when

$$\frac{d}{dt} = -v\frac{\partial}{\partial x}$$

and lead to equation

$$\left(1 - A^2v^2\right)\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0.$$

But if v is depending on t, we have

$$\frac{d}{dt} = \frac{\partial}{\partial t} - v\frac{\partial}{\partial x}.$$

If our value shall generally hold for x, it also must be

$$\frac{\partial X}{\partial t}\text{ small against }v\frac{\partial X}{\partial x}.$$

Now it is

$$\frac{\partial X}{\partial t} = \frac{\partial^2}{\partial x \partial t}U\left(1 - A^2v^2\right),$$

thus it must be

$$\frac{\partial}{\partial t}\left[\frac{\partial U}{\partial x}\left(1 - A^2v^2\right)\right]\text{small against }v\frac{\partial^2 U}{\partial x^2}\left(1 - A^2v^2\right),$$

or

$$A^2x\frac{\partial v}{\partial t}\text{ small against }1 - A^2v^2,$$

Also the values of Y,Z and M,N, give

$$\left[2x^2 - \left(1 - A^2v^2\right)\varrho^2\right]A^2\frac{\partial v}{\partial t}\text{ is small against }3x\left(1 - A^2v^2\right)$$

and

$$\left(1 - A^2v^2\right)\left[\left(x^2 + \left(1 - A^2v^2\right)\varrho^2\right)\right]\frac{\partial v}{\partial t}$$
$$- \left[2x^2 - \left(1 - A^2v^2\right)\varrho^2\right]A^2v^2\frac{\partial v}{\partial t}$$

must be small against $3x\left(1 - A^2v^2\right)v^2$.

This condition is fulfilled, when the dimensions of the space, in which the energy comes essentially into consideration, are sufficiently small. Because the terms to be neglected all contain the linear dimensions in a higher power. Though *dv* / *dt* may not be too great and the absolute velocity *v* not too small.

When this neglect is allowed, then we can put for the change of kinetic energy

$$\frac{d}{dt}\left(\frac{m}{2}v^2\right) = mv\frac{dv}{dt} = K\frac{dr}{dt}dt = m\frac{dr}{dt}\frac{d^2r}{dt^2},$$

where *K* denotes the electric force. In this manner, we have obtained the first and second law of motion of NEWTON.

Because when no external force is acting, the law of inertia is simply the law of conservation of electromagnetic energy, and the second law of NEWTON says here, that the work expended by the force during *dt*, is equal to the corresponding change of electromagnetic energy.

The third law of NEWTON, that maintains the equality of action and reaction, holds for all electrostatic forces between electric quanta. The mechanical forces must, for our stand point, identified with such forces. Since we make the assumption of a resting aether, this law doesn't hold for the general electromagnetic forces.

The theorem of parallelogram of forces is contained in our foundations in so far, as it holds for electric polarizations and for the forces acting between two electric quanta.

At last, as regards the rigid connections that can exist between several electric masses, those actually wouldn't exist from stand point. Only forces can arise, that are mutually in equilibrium. For example, if a pendulum swings, gravity is acting in a stretching way on the pendulum string, until the electric forces produced became equally great. Such forces expending no work, are to be introduced into the known Lagrangian form.

One can describe the foundation of mechanics sketched here, as diametrically opposed to that of HERTZ. The rigid connections, belonging to the presuppositions according to HERTZ, here show up as the action of complicated individual forces. Also the law of inertia is a comparably late consequence from the electromagnetic presuppositions. While HERTZ's mechanics is obviously aimed at presenting the electromagnetic equations as consequences, the relation is directly reversed here. With respect to logical form, the electromagnetically founded mechanics of course cannot compare itself to HERTZ's one, just because the system of MAXWELL's differential equations has found no precise-critical treatment at all, but it has, as it seems to me, a very remarkable advantage, namely (as it was shown) that it transcends ordinary mechanics, which therefore is only to be described as a first approximation. By that, the possibility is given, to decide for or against it by experience.

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